21 cm Signal Processing

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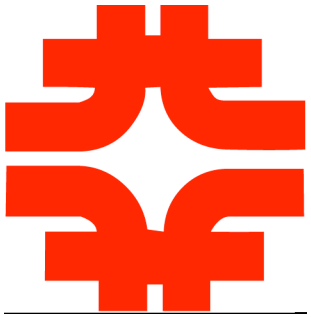
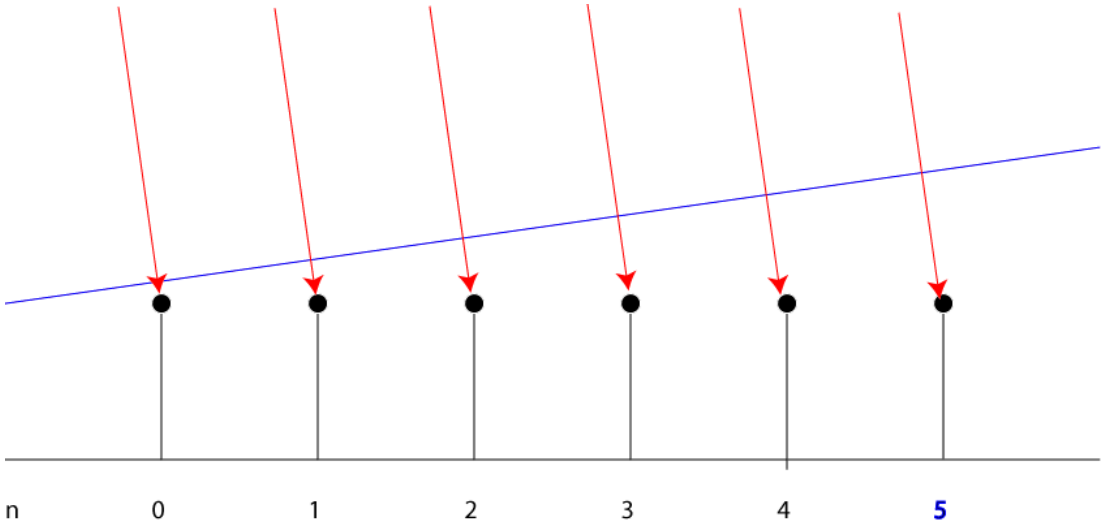
April 9, 2009



FFT Concept



$$\phi = nkd\theta$$

$$a_m = \frac{1}{N}\sum_{n=1}^{n} v(\theta)e^{in(\theta kd - 2\pi m/N)}$$

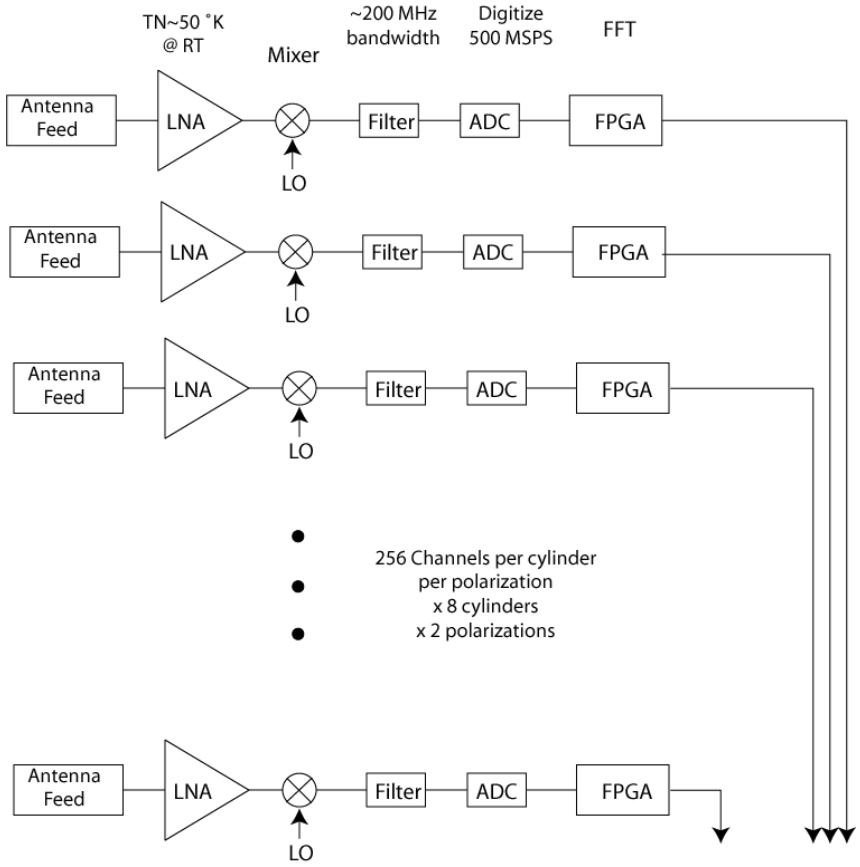
More FFT Concepts

$$|d_n|^2 = \frac{1}{N^2}\sum_{\ell=1}^{N}\sum_{m=1}^{N} e^{2\pi i(\ell-m)/N} a_\ell a_m^*$$

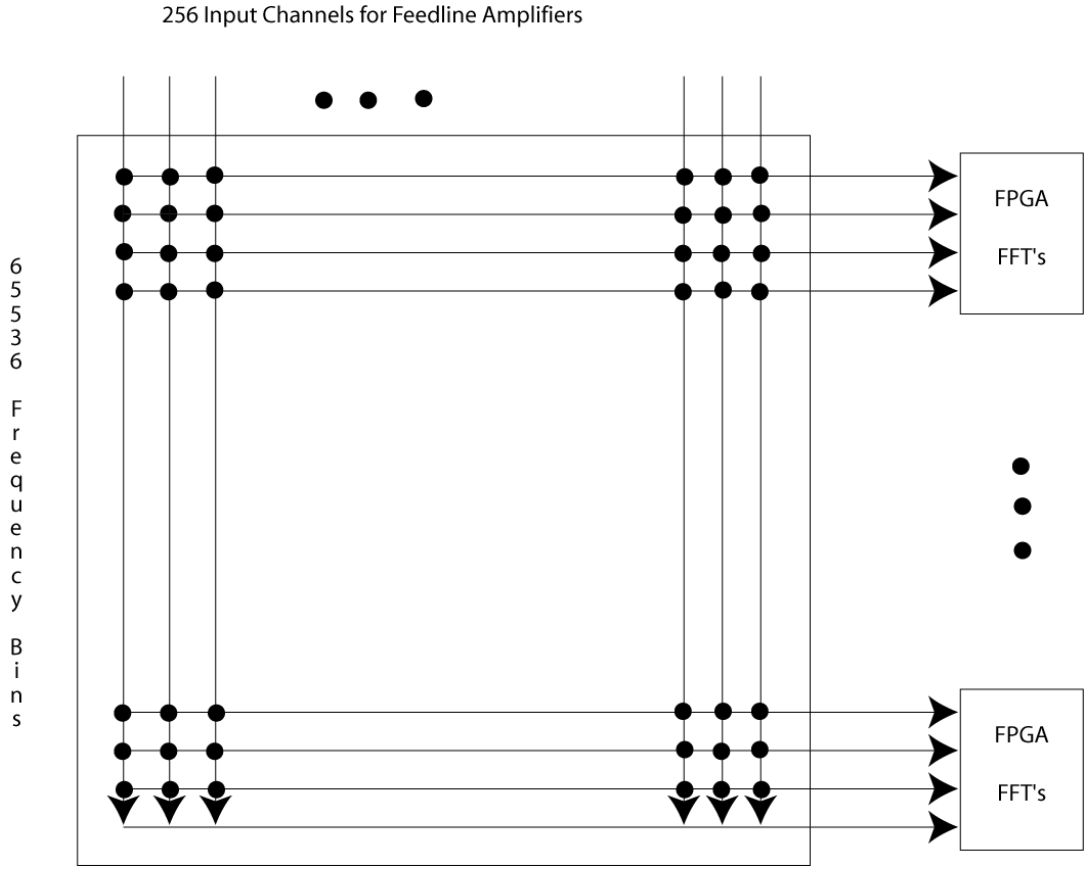
$$P = \sum_{m=1}^{N} a_m a_m^*$$

$$|d'_n|^2 = \frac{1}{N^2}\sum_{\ell=1}^{N}\sum_{m=1}^{N} e^{2\pi i(\ell-m)/N} a_\ell a_m^* - P$$

Signal Processing
First Stage



Signal Processing
Second Stage



Signal Processing
Third Stage

- Since there are only 8 cylinders, there are only 28 cross-correlation pairs.
  - It might be simpler to calculate the correlations by brute force for this small a number.
  - One might still want to do beam forming in firmware and save only the 8 independent beams
- One probably wants to save some individual correlations for calibration and debugging: both at stage 2 and stage 3.

Why FFT?

- It's the cheapest approach that will do the job.
  - Computationally efficient
  - Efficient storage (1 quantity per beam)
- The disadvantage is lack of flexibility
  - Dead channels or unstable channels
  - RFI rejection or any decision making algorithm

Analysis

- Make raw pixel map (average of 3 dimensional FFT power spectrum)
- Fit and remove foreground spectra on a pixel-by-pixel basis
- Apply inverse instrument model to get a corrected sky map
- Compute power spectrum of 21 cm
- Extract BAO signal

FFT Details

- There is some ambiguity as to what is meant by an FFT.
- It should be straight-forward to apply a window function (a.k.a. appodization) to any or all the FFT's.
- Equalization (weighting the FFT output) should also be straight-forward and presumably necessary to match channel gain functions.
- Poly-phase technology is also possible but is more computationally intensive and also results in some loss of signal.

Hardware Implementation

- FPGA's
- GPU's
- Processors

Conclusion

- We don't have time or energy to explore all possible alternatives.
- What should we adopt as the consensus approach?
- Where do we want an explicit trade study of alternatives?